Paper Reference(s)

6674/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Level

Monday 16 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6674), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

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1. (*a*) Write down the value of the real root of the equation

$$x^3 - 64 = 0.$$

(1)

(*b*) Find the complex roots of $x^3 - 64 = 0$, giving your answers in the form $a + \mathrm{i}b$, where a and b are real.

(4)

(*c*) Show the three roots of $x^3 - 64 = 0$ on an Argand diagram.

(2)

2.

$$\mathrm{f}(x) = 4\cos x + \mathrm{e}^{-x}.$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ has a root α between 1.6 and 1.7

(2)

(*b*) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures.

(4)

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3. The complex number z is defined by

$$z = \frac{a + 2\mathrm{i}}{a - \mathrm{i}}, \qquad a \in \mathbb{R}, \quad a > 0.$$

Given that the real part of z is $\frac{1}{2}$, find

(*a*) the value of a,

(4)

(*b*) the argument of z, giving your answer in radians to 2 decimal places.

(3)

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4. (*a*) Find, in terms of k, the general solution of the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = kt + 5, \quad \text{where } k \text{ is a constant and } t > 0.$$

(7)

For large values of t, this general solution may be approximated by a linear function.

(*b*) Given that $k = 6$, find the equation of this linear function.

(2)

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5. (*a*) Find, in the simplest surd form where appropriate, the exact values of x for which

$$\frac{x}{2} + 3 = \left|\frac{4}{x}\right|.$$

(5)

(*b*) Sketch, on the same axes, the line with equation $y = \frac{x}{2} + 3$ and the graph of $y = \left|\frac{4}{x}\right|$, $x \neq 0$.

(3)

(*c*) Find the set of values of x for which $\frac{x}{2} + 3 > \left|\frac{4}{x}\right|$.

(2)

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6. (*a*) Express $\dfrac{2}{(r+1)(r+3)}$ in partial fractions.

(2)

(*b*) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(an+b)}{6(n+2)(n+3)},$$

where a and b are constants to be found.

(6)

(*c*) Find the value of $\displaystyle\sum_{r=21}^{30} \frac{2}{(r+1)(r+3)}$, to 5 decimal places.

(3)

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7. (*a*) Show that the substitution $y = vx$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{y} + \frac{3y}{x}, \quad x > 0, \quad y > 0 \qquad \text{(I)}$$

into the differential equation

$$x\frac{\mathrm{d}v}{\mathrm{d}x} = 2v + \frac{1}{v}. \qquad \text{(II)}$$

(3)

(*b*) By solving differential equation (II), find a general solution of differential equation (I) in the form $y = \mathrm{f}(x)$.

(7)

Given that $y = 3$ at $x = 1$,

(*c*) find the particular solution of differential equation (I).

(2)

8.

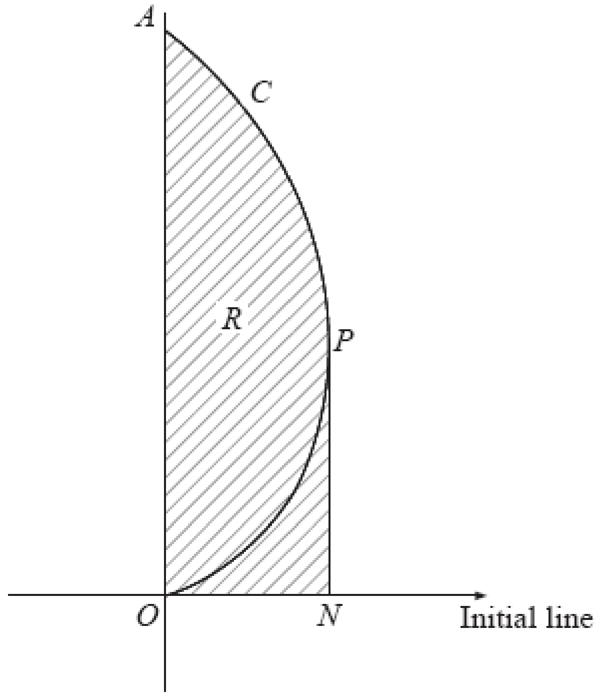


Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 4(1 - \cos\theta), \qquad 0 \le \theta \le \frac{\pi}{2}.$$

At the point P on C, the tangent to C is parallel to the line $\theta = \frac{\pi}{2}$.

(*a*) Show that P has polar coordinates $\left(2, \frac{\pi}{3}\right)$.

(5)

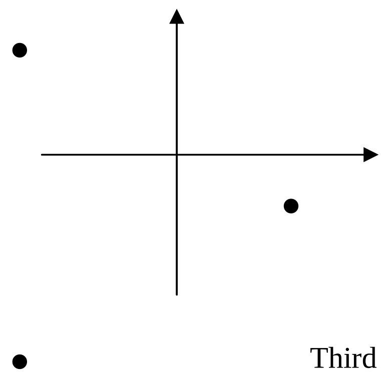
The curve C meets the line $\theta = \frac{\pi}{2}$ at the point A. The tangent to C at P meets the initial line at the point N. The finite region R, shown shaded in Figure 1, is bounded by the initial line, the line $\theta = \frac{\pi}{2}$, the arc AP of C and the line PN.

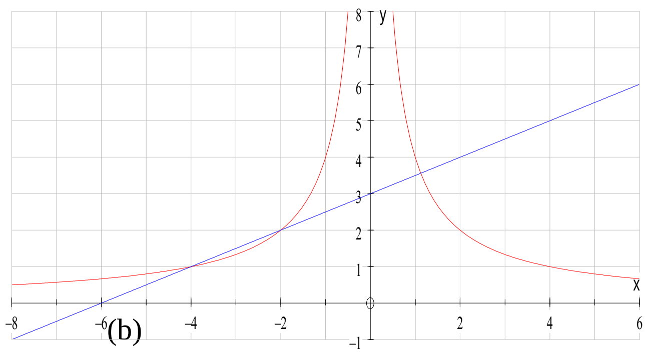
(*b*) Calculate the exact area of R.

(8)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (*a*)	4	B1 (1)
(*b*)	$(x-4)(x^2+4x+16)$	M1 A1
	$x=\dfrac{-4\pm\sqrt{16-64}}{2}$, $\quad x=-2\pm2\sqrt{3}\,\mathrm{i}$ $\quad$ (or equiv. surd for $2\sqrt{3}$)	M1 A1 (4)
(*c*)	 Root on + ve real axis, one other in correct quad.	B1
	Third root in conjugate complex position	B1 ft (2)
		(7 marks)
2. (*a*)	$\mathrm{f}(1.6)=...$ $\quad \mathrm{f}(1.7)=...$ $\quad$ (Evaluate both)	M1
	0.08… (or 0.09), $\quad$ –0.3… $\quad$ One +ve, one **–**ve or sign change, $\therefore$ root	A1 (2)
(*b*)	$\mathrm{f}\,'(x)=-4\sin x-\mathrm{e}^{-x}$	B1
	$1.6-\dfrac{\mathrm{f}(1.6)}{\mathrm{f}\,'(1.6)}$	M1
	$=1.6-\dfrac{4\cos1.6+\mathrm{e}^{-1.6}}{(-4\sin1.6-\mathrm{e}^{-1.6})}$ $\quad \left(=1.6-\dfrac{0.085...}{-4.2...}\right)$	A1
		A1 (4)
		(6 marks)
3. (*a*)	$z=\dfrac{(a+2\mathrm{i})(a+\mathrm{i})}{(a-\mathrm{i})(a+\mathrm{i})}=\dfrac{a^2+3a\mathrm{i}-2}{a^2+1}$	M1 A1
	$\dfrac{a^2-2}{a^2+1}=\dfrac{1}{2}$, $\quad 2a^2-4=a^2+1$ $\quad a=\sqrt{5}$ $\quad$ (presence of $-\sqrt{5}$ also is A0)	M1 A1 (4)
(*b*)	Evaluating their “$\dfrac{3a}{a^2+1}$”, or “ $3a$” $\left(\dfrac{\sqrt{5}}{2}\text{ or } 3\sqrt{5}\right)$ $\quad$ (ft errors in part *a*)	B1 ft
	$\tan\theta=\dfrac{3a}{a^2-2}$ $\quad \left(=\dfrac{3\sqrt{5}}{3}\right)$, $\quad \arg z = 1.15$ $\quad$ (accept answers which round to 1.15)	M1 A1 (3)
		(7 marks)

Question Number	Scheme	Marks
4. (*a*)	$m^2 + 4m + 3 = 0$ $\quad m = -1,\ m = -3$	M1 A1
	C.F. $(x =)Ae^{-t} + Be^{-3t}$ $\quad$ must be function of *t*, not *x*	A1
	P.I. $x = pt + q$ $\quad$ **(or** $\quad x = at^2 + bt + c$)	B1
	$4p + 3(pt + q) = kt + 5$ $\quad 3p = k$ $\quad$ (Form at least one eqn. in *p* and/or *q*)	M1
	$4p + 3q = 5$	
	$p = \frac{k}{3}$, $\quad q = \frac{5}{3} - \frac{4k}{9}\ \left(= \frac{15-4k}{9}\right)$	A1 (5)
	General solution: $x = Ae^{-t} + Be^{-3t} + \frac{kt}{3} + \frac{15-4k}{9}$ (must include *x* = and be function of *t*)	A1 ft
(*b*)	When $k = 6$, $\quad x = 2t - 1$	M1
		A1 cao (2)
5. (*a*)	$\frac{4}{x} = \frac{x}{2} + 3$ $\quad x^2 + 6x - 8 = 0$ $\quad x = ...,\ \left(\frac{-6 \pm \sqrt{68}}{2}\right)$ $\quad -3 \pm \sqrt{17}$	M1 A1
	$-\frac{4}{x} = \frac{x}{2} + 3,$ $\quad x^2 + 6x + 8 = 0$ $\quad x = -4$ and -2	M1 A1
	Three correct solutions (and no extras): $-4,\ -2,\ -3 + \sqrt{17}$	A1 (5)
(*b*)	 (b)	
	Line through point on –ve x axis and + y axis	B1
	Curve	B1
	Intersections in correct quadrants	B1 (3)
(*c*)	$-4 < x < -2,$ $\quad x > -3 + \sqrt{17}$ o.e.	M1 A1 (2)
		(10 marks)

Question Number	Scheme	Marks
6. (*a*)	$\frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3}$ M: $\frac{2}{(r+1)(r+3)} = \frac{A}{r+1} + \frac{B}{r+3}$	M1 A1 (2)
(*b*)	$r = 1$: $\left(\frac{2}{2\times 4}\right) = \frac{1}{2} - \frac{1}{4}$	M1
	$r = 2$: $\left(\frac{2}{3\times 5}\right) = \frac{1}{3} - \frac{1}{5}$	
	$r = n-1$: $\left(\frac{2}{n(n+2)}\right) = \frac{1}{n} - \frac{1}{n+2}$	
	$r = n$: $\left(\frac{2}{(n+1)(n+3)}\right) = \frac{1}{n+1} - \frac{1}{n+3}$	A1
	Summing: $\sum = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$	M1 A1
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{6(n+2)(n+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$ cso	M1 A1 (6)
(*c*)	$\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{20} = \frac{30\times 163}{6\times 32\times 33} - \frac{20\times 113}{6\times 22\times 23}$, $= 0.02738$	M1 A1 A1 (3)
		(10 marks)
7. (*a*)	$\frac{dy}{dx} = v + x\frac{dv}{dx}$	B1
	$\left(v + x\frac{dv}{dx}\right) = \frac{x}{vx} + \frac{3vx}{x} \Rightarrow x\frac{dv}{dx} = 2v + \frac{1}{v}$	M1 A1 (3)
(*b*)	$\int \frac{v}{1+2v^2}\,dv = \int \frac{1}{x}\,dx$	M1
	$\frac{1}{4}\ln(1+2v^2), \quad = \ln x \ (+C)$	M1 A1 B1
	$Ax^4 = 1 + 2v^2$	M1
(*c*)	$Ax^4 = 1 + 2\left(\frac{y}{x}\right)^2$ so $y = \sqrt{\frac{Ax^6 - x^2}{2}}$ or $y = x\sqrt{\frac{Ax^4 - 1}{2}}$ or $y = x\sqrt{\left(\frac{1}{2}e^{4\ln x + 4c} - \frac{1}{2}\right)}$	M1 A1 (7)
	$x = 1$ at $y = 3$: $3 = \sqrt{\frac{A-1}{2}}$ $A = \ldots$	M1
(*d*)	$y = \sqrt{\frac{19x^6 - x^2}{2}}$ or $y = x\sqrt{\frac{19x^4 - 1}{2}}$	A1 (2)
		(12 marks)

Question Number	Scheme	Marks
8. (*a*)	$r\cos\theta = 4(\cos\theta - \cos^2\theta)$ or $r\cos\theta = 4\cos\theta - 2\cos 2\theta - 2$	B1
	$\dfrac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta} = 4(-\sin\theta + 2\cos\theta\sin\theta)$ or $\dfrac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta} = 4(-\sin\theta + \sin 2\theta)$	M1 A1
	$4(-\sin\theta + 2\cos\theta\sin\theta) = 0 \Rightarrow \cos\theta = \dfrac{1}{2}$ which is satisfied by $\theta = \dfrac{\pi}{3}$ and $r = 2$(*)	M1 A1 (5)
(*b*)	$\dfrac{1}{2}\int r^2\,\mathrm{d}\theta = (8)\int\left(1 - 2\cos\theta + \cos^2\theta\right)\mathrm{d}\theta$	M1
	$= (8)\left[\theta - 2\sin\theta + \dfrac{\sin 2\theta}{4} + \dfrac{\theta}{2}\right]$	M1 A1
	$= 8\left[\dfrac{3\theta}{2} - 2\sin\theta + \dfrac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi/2} = 8\left(\left(\dfrac{3\pi}{4} - 2\right) - \left(\dfrac{\pi}{2} - \sqrt{3} + \dfrac{\sqrt{3}}{8}\right)\right) = 2\pi - 16 + 7\sqrt{3}$	M1
	Triangle: $\dfrac{1}{2}(r\cos\theta)(r\sin\theta) = \dfrac{1}{2}\times 1\times\sqrt{3} = \dfrac{\sqrt{3}}{2}$	M1 A1
	Total area: $\left(2\pi - 16 + 7\sqrt{3}\right) + \dfrac{\sqrt{3}}{2} = \left(2\pi - 16\right) + \dfrac{15\sqrt{3}}{2}$	A1 A1 (8)
		(13 marks)